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Late Times Accelerations Viewed Through an LRS Bianchi Type I Cosmological Model

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Abstract Through a suitable choice of the Hubble parameter a cosmological model of LRS Bianchi type I is obtained in which late times accelerations of the expanding universe follow naturally. The time of transition from decelerated phase to accelerated one is also realized.

Keywords Accelerated phase in expanding universe · LRS Bianchi type-I space-time · Variations in G and Λ

1 Introduction

That our evolving universe is passing presently through an accelerated phase is indicated by observations made recently in last few years [2, 3, 5, 6]. This is somewhat amazing in the traditionally held picture of a big-bang model in which expansion of the universe has usually been conceived to decelerate with time. Obviously then, there must have been a transition from decelerated phase to accelerated one in the evolution process of the universe. This phase transition could have been due to dominance [4] of dark energy (represented by cosmological (constant) term Λ) over other kinds of matter fields (collectively represented by gravitational (constant) term G). Hence, consideration of variations in Λ and G, so that they are not mere constants, may reveal the expected dominance. We, therefore, consider in this paper, each of Λ and G, which appears in Einstein's field equations, as variable. Also, following the lines floated by [1], we consider a suitable functional form of the Hubble parameter H whereby accelerations as well as decelerations are both possible to appear in due course during the evolution of the universe.

The underlying space-time that we consider is of LRS (Locally Rotationally Symmetric) Bianchi type I which is the simplest generalization (corresponding to zero curvature of spatial sections) of that of FRW (Friedmann Robertson Walker) model. The space-time is filled with a perfect fluid distribution throughout, and usual conservation equations are taken into account. The resulting model is found to account for the accelerations satisfactorily.

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2 Derivation of the Model

We consider the space-time given by an LRS Bianchi type I metric:

$$ds^2 = -dt^2 + A^2(x,t)dx^2 + B^2(x,t)(dy^2 + dz^2), \tag{1}$$

which is filled with a co-moving perfect fluid, whose energy-momentum tensor T_{ij} is given by

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} \tag{2}$$

with $v_i v^i = -1$; ρ and p respectively being energy density and pressure of the fluid. We take the Einstein's field equations in the form

$$R_{ij} - \frac{1}{2}R g_{ij} = -8\pi G(x,t)T_{ij} + \Lambda(x,t) g_{ij}, \tag{3}$$

which, in view of (1) and (2), lead to

$$8\pi G p = -\left[2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2\right] + \frac{1}{A^2}\left(\frac{B_1}{B}\right)^2 + \Lambda \tag{4}$$

$$= -\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB}\right] + \frac{1}{A^2}\left[\frac{B_{11}}{B} - \frac{A_1 B_1}{AB}\right] + \Lambda \tag{5}$$

$$8\pi G\rho = \left[\frac{2A_4 B_4}{AB} + \left(\frac{B_4}{B}\right)^2\right] + \frac{1}{A^2}\left[-\frac{2B_{11}}{B} + \frac{2A_1 B_1}{AB} - \left(\frac{B_1}{B}\right)^2\right] - \Lambda \tag{6}$$

and

$$0 = \frac{B_{14}}{B} - \frac{A_4 B_1}{AB}, \tag{7}$$

in which the suffixes 1 and 4 stand for differentiation with respect to x and t respectively.

Also, vanishing divergence of T_{ij} leads to

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = 0 \tag{8}$$

whereas that of R.H.S. of (3) leads to

$$\Lambda_4 + 8\pi\left[G_4\rho + G\left\{\rho_4 + (\rho + p)\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\right\}\right] = 0 \tag{9}$$

and

$$-\Lambda_1 + 8\pi(G_1 p + G p_1) = 0. \tag{10}$$

Equation (7) gives on integration

$$B_1 = Af, \tag{11}$$

f being an arbitrary function of x. From (4), (5) and (11) we obtain

$$\frac{B}{B_1}\left(\frac{B_{44}}{B}\right)_1 + \frac{B_4}{B_1}\left(\frac{B_1}{B}\right)_4 + \frac{f^2}{B^2}\left(1 - \frac{B}{B_1}\frac{f_1}{f}\right) = 0 \tag{12}$$

which is satisfied for

$$B = fF, \tag{13}$$

where F is an arbitrary function of t.

Thus, in view of (11) and (13), the line-element (1) can be written as

$$ds^2 = -dt^2 + F^2[d\bar{x}^2 + e^{2\bar{x}}(dy^2 + dz^2)], \tag{14}$$

where $\bar{x} = \log f$.

From (14) it is obvious that F is the scale factor of the model under investigation.

The set of equations (4)–(10), now reduces to

$$8\pi Gp = \Lambda - \left[2\frac{F_{44}}{F} + \left(\frac{F_4}{F}\right)^2\right] + \frac{1}{F^2} \tag{15}$$

$$8\pi G\rho = -\Lambda + 3\left(\frac{F_4}{F}\right)^2 - \frac{3}{F^2} \tag{16}$$

$$\Lambda_4 + 8\pi\left[G_4\rho + G\left\{p_4 + 3(\rho + p)\frac{F_4}{F}\right\}\right] = 0 \tag{17}$$

and

$$p_4 + 3(\rho + p)\frac{F_4}{F} = 0. \tag{18}$$

We assume that the fluid satisfies the barotropic equation of state, viz.,

$$p = w\rho, \quad 0 \le w \le 1. \tag{19}$$

Equation (18) then gives on integration

$$\rho = kF^{-3(w+1)}, \tag{20}$$

where k is a positive constant. Therefore, from (15), (16) and (20), we get

$$8\pi G = -\frac{2}{k(w+1)}F^{3(w+1)}\left[\left(\frac{F_4}{F}\right)_4 + \frac{1}{F^2}\right] \tag{21}$$

and

$$\Lambda = \frac{2}{(w+1)}\left(\frac{F_4}{F}\right)_4 + 3\left(\frac{F_4}{F}\right)^2 - \left(\frac{3w+1}{w+1}\right)\frac{1}{F^2}. \tag{22}$$

We find that (17) is satisfied identically by (20), (21) and (22). Thus each of ρ (and hence p), G and Λ is expressed in terms of F and its derivatives.

Further, for the determination of F, we follow the lines set by Ellis and Madsen [1] in which both the decelerating and the accelerating phases are accountable in an expanding universe model. Accordingly, we take the Hubble parameter H in the form

$$H(F) \equiv \frac{F_4}{F} = a(F^{-n} + 1), \tag{23}$$

where a (>0) and n (>1) are constants, which put the deceleration parameter q in the form

$$q \equiv \frac{-FF_{44}}{(F_4)^2} = \frac{n}{F^n+1} - 1. \tag{24}$$

Equation (23) yields

$$F^n = e^{na(t+t_0)} - 1, \tag{25}$$

in which t_0 is constant of integration, which we set to be zero without any qualitative loss. Thus F in the model turns out to be

$$F^n = e^{nat} - 1. \tag{26}$$

3 Discussions

Corresponding to F given by (26), the model (14) is found to have

$$\text{Shear} = \sigma = 0 \text{ (identically)},$$

$$\text{Expansion scalar} = \Theta \equiv v^i_{;i} = 3\frac{F_4}{F} = 3H = \frac{3ae^{nat}}{e^{nat}-1},$$

the semi-colon standing for covariant differentiation;

$$\rho = k(e^{nat}-1)^{-3(w+1)/n},$$

$$\Lambda = -\frac{2a^2n}{w+1}\left[\frac{e^{nat}}{(e^{nat}-1)^2}\right] + \frac{3a^2e^{2nat}}{(e^{nat}-1)^2} - \frac{3w+1}{w+1}\left[\frac{1}{(e^{nat}-1)^{2/n}}\right],$$

$$8\pi G = -\frac{2}{k(w+1)}(e^{nat}-1)^{3(w+1)/n}\left[-\frac{a^2ne^{nat}}{(e^{nat}-1)^2} + \frac{1}{(e^{nat}-1)^{2/n}}\right].$$

We note that Θ tends to ∞ and $3a$ when $t \to 0$ and ∞ respectively. Also, ρ tends to ∞ when $t \to 0$, and it tends to zero when $t \to \infty$. So, the model starts with a big-bang at $t = 0$ and continues expanding afterwards till $t = \infty$ with material content vanishing there. Isotropy prevails throughout, for $\sigma/\Theta = 0$ identically in the model. The deceleration parameter $q \to n - 1$ when $t \to 0$ whereas it tends to -1 when $t \to \infty$. For matter dominated case ($w = 0$), we find, for $n = 2$ and $a^2 = \frac{1}{2}$, that $G \to \infty$ and 0 whereas $\Lambda \to -\infty$ and 3/2; respectively when $t \to 0$ and ∞. Thus a positive Λ (employing repulsion) and a negative q (employing acceleration) for large t indicate late times accelerations. This is associated with relative dominance of Λ over G, for $\Lambda/G \to \infty$ when $t \to \infty$. Also, we have

$$q = 0 \quad \text{if } e^{\sqrt{2}t} = 2 \quad \Longrightarrow \quad t = \frac{1}{\sqrt{2}}\log 2 \equiv T \text{ (say)}$$

At $t = T$, we get $\Lambda = 1$ and $G = \frac{1}{4\pi k}$ so that $\frac{\Lambda}{G} = 4\pi k = 1$ if $k = \frac{1}{4\pi}$.

But, Λ/G, being an increasing function of t, implies $\Lambda > G$ for $t > T$ for this k. So, the acceleration could have started just after $t = T$ when Λ also has started dominating over G. Prior to $t = T$, there were decelerations only; and at $t = T$ itself, neither deceleration nor acceleration was there.

In the radiation dominated case ($w = 1/3$), we find, for these specific values of the constants, that Λ and G cease to vary for ever, each becoming some constant. The model can be discussed for other possible values of the constants.

4 Conclusions

With the solution at hand we analyse the situations with passage of time t. For some specific values of the constants appearing in the model, we encounter a big-bang start of the universe whose further expansion consists of early decelerations and late times accelerations. The transition from decelerated phase to accelerated one is realized enroute during the evolution process.

Such a view encompassing the observed accelerations of the expanding universe has been an outcome mostly of a scale factor obtained by a suitable choice of the Hubble parameter. Specification of the scale factor otherwise may explain other interesting features of the universe.

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